Indian Statistical Institute
First Semester 2006-2007
Mid-Semester Examination
M.Math.II Year
Graph Theory and Combinatorics
Time: 3 hrs   Date:15-09-06   Max. Marks : 100
Instructor: N S N Sastry

<u>Note:</u> Answers should be complete and to the point. Answer all questions.

1. Define the extendability of a $2-(v, k, \lambda)$ design. Show that a symmetric $2-(n^2+n+1, n+1, 1)$ design extends only if $(n+2)$ divides 12. Show that a projective plane of order 2 is unique up to isomorphism and admits an extension. [20]

2. Determine the set of all cyclic codes of length 8 over $\mathbb{F}_3$. Compute the parameters of the cyclic code over $\mathbb{F}_3$ of length 8 generated by $(X^2 + 1)(X^2 + X + 2)$. [20]

3. Compute the parameters of the $t-(v, k, \lambda)$ design $(P_0^n(q), P_k^n(q))$, for the largest possible t. Here, $P_0^n(q)$ denotes the set of all points of a projective space of dimension n over $\mathbb{F}_q$, $P_k^n(q)$ denotes the set of its k dimensional subspaces and $2 < k < n$. [15]

4. For a set S containing n elements, $n > 0$, determine the number of permutations of S which do not fix any element of S. [10]

5. Define a Hadamard matrix. Show that the Kronecker product of two Hadamard matrices is a Hadamard matrix. Deduce that there exists a Hadamard matrix of order 2^n for each n. [1+8+1]

6. If $(X, \mathbb{B})$ is a $2-(v, k, \lambda)$ design, then show that $(X, \{X \backslash B : B \in \mathbb{B}\})$ also is a 2-design. Compute its parameters. [10]

7. Compute the parameters of the binary cyclic code of length n, n-odd, generated by the minimal polynomial over $\mathbb{F}_2$ of a primitive nth-root of 1. [20]

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